**CS3431-A22 Wong**

**Assignment 5: Solutions**

**Problem 1 (50 Points)**

For the relational schema given below and its corresponding functional dependencies (FDs):

R(A, B, C, D, E) S = { AB → D,   AB → E, BC → D, C → A, CD → B } answer the following questions:

1. (10 points) Find all candidate keys of the relation through an exhaustive set of attribute closures. Specify when an attribute set closure is trivial, or a super key – just enter “super”.

Calculation of the candidate keys are shown below:

A+ = trivial AB+ = ABDE BD+ = trivial

B+ = trivial AC+ = trivial BE+ = trivial

C+ = AC AD+ = trivial CD+ = ABCDE

D+ = trivial AE+ = trivial CE+ = ACE

E+ = trivial BC+ = ABCDE DE+ = trivial

ABC+ = super ADE+ = trivial ABCD+ = super

ABD+ = ABDE BCD+ = super ABCE+ = super

ABE+ = ABDE BCE+ = super ABDE+ = trivial

ACD+ = super BDE+ = trivial ACDE+ = super

ACE+ = trivial CDE+ = super BCDE+ = super

Candidate keys: BC and CD

1. (5 points) Find the FDs in order that they appear in S that violate BCNF.

The following FDs violate BCNF: AB 🡪 D, AB 🡪 E, C 🡪 A because the left sides are not a super key.

1. (15 Points) Decompose the relations to satisfy BCNF using the **right-most** FD violation. For example, if C 🡪 A and CD 🡪 B are in BCNF but the other FDs are in violation, you would use BC 🡪 D for the decomposition. Specify which FD is used to make the decomposition. If there is a multi-step decomposition, then indicate each step along with which FD is used for the decomposition.

S = { AB → D,   AB → E, BC → D, C → A, CD → B }

Using C 🡪 A as the violation,

C+ = AC therefore R1(A, C) and R2(B, C, D, E).

R1 is a 2-attribute relation so it is in BCNF.

Find attribute closures for all combinations of R2 to find the candidate keys:

B+ = trivial BC+ = BCDE CD+ = BCDE BCD+ = super

C+ = trivial BD+ = trivial CE+ = trivial BCE+ = super

D+ = trivial BE+ = trivial DE+ = trivial BDE+ = trivial

E+ = trivial CDE+ = super

Super keys are BC, BCD, BCE, CD, CDE

Candidate keys are BC and CD

From the closures above we get the following functional dependencies:

BC 🡪 D

BC 🡪 E

CD 🡪 B

CD 🡪 E

BCD 🡪 E remove because of CD 🡪 E

BCE 🡪 D remove because of BC 🡪 D

CDE 🡪 B remove because of CD 🡪 B

BC and CD are the candidate keys.

The functional dependencies for R2 are BC 🡪 D, BC 🡪 E, CD 🡪 E, CD 🡪 B. Since all of the left sides are candidate keys, R2 is in BCNF.

The decomposed BCNF is R1(A, C) and R2(B, C, D, E).

Notice that because some of the FDs are prime, not all of the original functional dependencies are retained.

1. (10 points) Calculate the minimal basis, S’, for R. Sort the FDs in order alphabetically by the left side. As an example, { D 🡪 C, DE 🡪 B, A 🡪 E } would become { A 🡪 E, D 🡪 C, DE 🡪 B }.

S = { AB → D,   AB → E, BC → D, C → A, CD → B }

Test removal of FDs

* 1. If AB 🡪 D is removed from the FDs, then AB+ = ABE. Since D is not in the closure, this FD cannot be removed.
  2. If AB 🡪 E is removed from the FDs, then AB+ = ABD. Since E is not in the closure, this FD cannot be removed.
  3. If BC 🡪 D is removed from the FDs, then BC+ = ABCDE. Since D is in the closure, this FD CAN be removed because it is REDUNDANT!

S’ = { AB → D,   AB → E, C → A, CD → B }

* 1. If C 🡪 A is removed from S’ FDs, then C+ = C. Since A is not in the closure, this FD cannot be removed.
  2. If CD 🡪 B is removed from S’ FDs, the CD+ = ACD. Since B is not in the closure, this FD cannot be removed.

S’ = { AB → D,   AB → E, C → A, CD → B }

Now test to see if any attributes may be removed.

* 1. A+ = A. Therefore, A 🡪 D does not exist. B cannot be removed from AB 🡪 D.
  2. A+ = A. Therefore, A 🡪 E does not exist. B cannot be removed from AB 🡪 E.
  3. B+ = B. Therefore, B 🡪 D does not exist. A cannot be removed from AB 🡪 D.
  4. B+ = B. Therefore, B 🡪 E does not exist. A cannot be removed from AB 🡪 E.
  5. C+ = AC. Therefore, C 🡪 B does not exist. D cannot be removed from CD 🡪 B.
  6. D+ = D. Therefore, D 🡪 B does not exist. C cannot be removed from CD 🡪 B.

The minimal basis for S is

S’ = { AB → D,   AB → E, C → A, CD → B }

1. (5 points) Using S’ instead of S for the rest of this problem, find the FDs in order that they appear in S’ that violate 3NF.

Since the candidate keys are BC and CD (same as in S):

AB 🡪 D is prime and CD 🡪 B is in 3NF because CD is a key

The following FDs violate 3NF because the left sides are not super keys nor the right sides prime:

AB 🡪 E, C 🡪 A

1. (5 points) Decompose R into 3NF.
   1. Group together FDs with the same left side: AB 🡪 DE, C 🡪 A, CD 🡪 B
   2. R1(A, B, D, E) R2(A, C) R3(B, C, D) since the candidate key BC is in R3, there is no need to add another relation.

**Problem 2 (50 Points)**

Given a relation R(A, B, C, D, E) with functional dependencies S = { BE 🡪 A, CD 🡪 E, AD 🡪 C }

1. (10 points) List all of the candidate keys of R. Show all of the attribute closures! Note when an attribute set closure is trivial or when it is a super key – indicate “super”.

A+ = trivial AB+ = trivial BC+ = trivial ABC+ = trivial BCD+ = super BCDE+ = super

B+ = trivial AC+ = trivial BD+ = trivial ABD+ = super BCE+ = ABCE ACDE+ = trivial

C+ = trivial AD+ = ACDE BE+ = ABE ABE+ = trivial BDE+ = super ABDE+ = super

D+ = trivial AE+ = trivial CD+ = CDE ACD+ = ACDE CDE+ = trivial ABCE+ = ABCE

E+ = trivial CE+ = trivial ACE+ = trivial ABCD+ = super

DE+ = trivial ADE+ = ACDE

Candidate keys for R are ABD, BCD, BDE

1. (5 points) List the dependencies, in the order given in S, that violate **BCNF**.

Since none of the left sides are superkeys, all three FDs violate BCNF:

BE 🡪 A, CD 🡪 E, AD 🡪 C

1. (15 points) Decompose the relations to satisfy BCNF using the **left-most** FD violation. For example, if BE 🡪 A is in BCNF but the other FDs are in violation, you would use CD 🡪 E for the decomposition. Specify which FD is used to make the decomposition. If there is a multi-step decomposition, then indicate each step along with which FD is used for the decomposition.

BCNF Violation: BE 🡪 A.

BE+ = ABE

Therefore, we decompose into R1(A, B, E) and R2(B, C, D, E)

For R1 calculate the attribute closures (using R’s FDs) to find the new candidate keys and FDs:

A+ = trivial AB+ = trivial

B+ = trivial AE+ = trivial

E+ = trivial BE+ = ABE

The candidate key for R1 is BE, and it has a single FD: BE 🡪 A. R1 is in BCNF.

For R2 calculate the attribute closures (using R’s FDs) to find the new candidate keys and FDs:

B+ = trivial BC+ = trivial CD+ = CDE BCD+ = BCDE

C+ = trivial BD+ = trivial CE+ = trivial BCE+ = trivial

D+ = trivial BE+ = trivial DE+ = trivial BDE+ = BCDE

E+ = trivial CDE+ = trivial

The functional dependencies are

The candidate keys are BCD and BDE. The functional dependencies are

CD 🡪 E

BCD 🡪 E remove because of CD 🡪 E

BDE 🡪 C

CD 🡪 E is still a BCNF Violation.

CD+ = CDE. We decompose R2 into R3(C, D, E) and R4(B, C, D)

For R3 calculate the attribute closures (using R’s FDs) to find the new candidate keys and FDs:

C+ = trivial CD+ = CDE

D+ = trivial CE+ = trivial

E+ = trivial DE+ = trivial

R3’s candidate key is CD and its only functional dependency is CD 🡪 E. R3 is in BCNF.

For R4 calculate the attribute closures (using R’s FDs) to find the new candidate keys and FDs:

B+ = trivial BC+ = trivial

C+ = trivial BD+ = trivial

D+ = trivial CD+ = trivial

Since R4 has only trivial FDs, it is in BCNF.

The final BCNF decomposition is

R1(A, B, E)

R3(C, D, E)

R4(B, C, D)

1. (10 points) Calculate the minimal basis, S’, for R. Sort the FDs in order alphabetically by the left side. As an example, { D 🡪 C, DE 🡪 B, A 🡪 E } would become { A 🡪 E, D 🡪 C, DE 🡪 B }.
   1. If BE 🡪 A is removed from the FDs, then BE+ = BE. Since A is not in the closure, this FD cannot be removed.
   2. If CD 🡪 E is removed from the FDs, then CD+ = CD. Since E is not in the closure, this FD cannot be removed.
   3. If AD 🡪 C is removed from the FDs, then AD+ = AD. Since C is not in the closure, this FD cannot be removed.

None of the FDs can be removed.

* 1. Because the closure of any single attribute is itself, non of the attributes on the left side of the FDs can be removed.

S’ = S because S is already in 3NF.

1. (10 points) If S’ is not in 3NF, decompose the relation R to satisfy 3NF. Otherwise state that S’ is in 3NF and why.

BE 🡪 A is prime

CD 🡪 E is prime

AD 🡪 C is prime

Since all FDs are prime, then S is in 3NF and does not need to be decomposed.